

~~FILE COPY~~

AD A100740

~~LEVEL~~

C

STATISTICAL PROPERTIES OF A RANDOM ARRAY
OF ACCUSTIC SENSORS IN A MULTIPATH ENVIRONMENT

DTIC
ELECTED
JUN 26 1981

(10) Fred Haber

APPROVED FOR PUBLIC RELEASE
DISTRIBUTION UNLIMITED

J. F. J. Haber Technical Library

100-101 1980

National Technical Library

Valley Forge Research Center
The Moore School of Electrical Engineering
University of Pennsylvania
Philadelphia, Pennsylvania

ABSTRACT

The statistical properties of the power pattern and main beam gain are here determined for an array of randomly located submerged acoustic sensors. The system investigated models a sparse array in which the sensors are dropped haphazardly over a region or in which the sensors, however they are initially placed, become spatially diffused by a process akin to a two-dimensional random walk. Signal energy is assumed to arrive over a vertically dispersive channel typical of the long range deep sea acoustic channel with a bigradient sound speed profile. Results are obtained for the mean value of the power pattern and the mean and variance of the main beam power gain as a function of array size. It is shown that in typical cases the mean array gain will preserve its value within 3 dB until the dispersion parameter of the array measured by the standard deviation of element location is around 35 wavelengths. The results are found to be consistent with results obtained by others for the coherence distance in a multipath acoustic field.

81 6 125 070

TABLE OF CONTENTS

INTRODUCTION	1
STATISTICAL PROPERTIES OF ARRAY POWER PATTERN . .	4
CONCLUSION	17
ACKNOWLEDGMENTS	19

Accession For	
NTIS GRA&I	<input type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By . . .	
Distribution/ . . .	
Availability Codes	
Dist	Avail and/or Special
A	

STATISTICAL PROPERTIES OF A RANDOM ARRAY OF
ACOUSTIC SENSORS IN A MULTIPATH ENVIRONMENT

INTRODUCTION

The statistical properties of the power pattern of an array of acoustic sensors suspended from individual freely floating buoys, receiving from a source via a time-dispersive medium are here investigated. The system geometry is indicated in Figure 1. The array elements, numbering N , are assumed distributed in a region centered on the origin of coordinates. By independent means the system learns the position (x_i, y_i, z_i) , $i = 1, 2, \dots, N$ of each of its elements. We suppose that the array then organizes itself at a specified frequency forming a beam aimed in some selected direction by adding suitably phased versions of the element outputs. If, for instance, the element outputs were of unit magnitude at the specified frequency, if the signal were arriving at angle (θ, ϕ) , and if the array were focused to

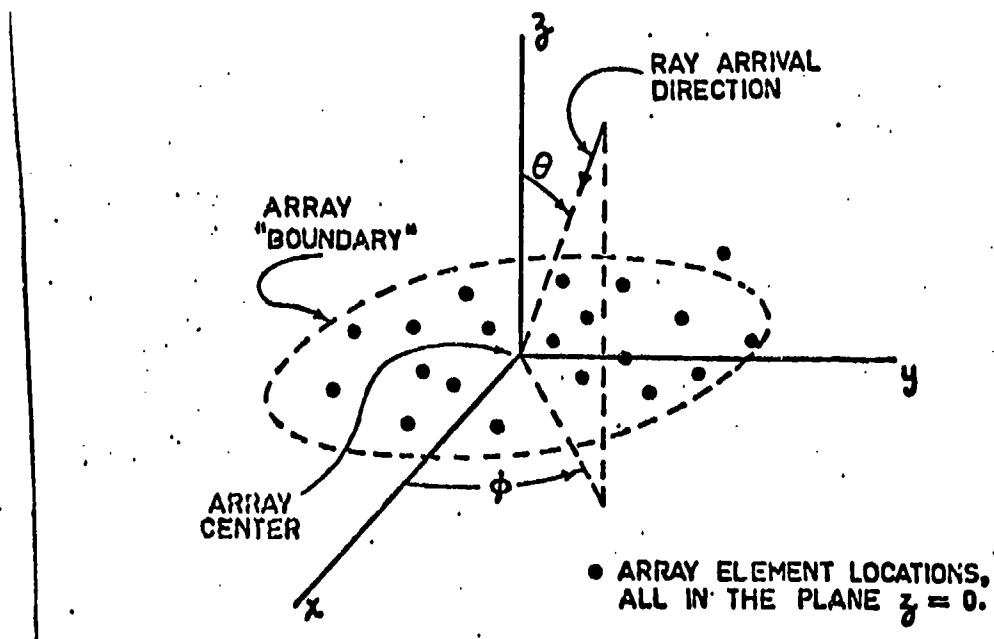


FIGURE 1. / ARRAY GEOMETRY

receive a signal ray arriving in the y - z plane at a colatitude angle θ_s , the complex array output at that frequency would be,

$$A = \sum_{n=1}^N e^{jk[x_n \sin\theta \cos\phi + y_n (\sin\theta \sin\phi - \sin\theta_s) + z_n (\cos\theta - \cos\phi_s)]} \quad (1)$$

where the wavenumber $k = 2\pi/\lambda$, λ being the wavelength. In this work we will assume that all elements are at the same depth, all in the x - y plane, so that all $z_n = 0$. Though surface waves will cause vertical displacement of the elements, the wavelengths of interest are such that in placid seas (sea state ≤ 4) the displacement is less than 0.1 wavelength. Furthermore, the system envisioned is expected to utilize a drouge with each buoyed element so that vertical motion will be filtered. Element positions in the (x, y) plane will be assumed independently distributed according to some appropriate two-dimensional probability density function (pdf) $f_{XY}(x, y)$, as will be discussed later.

The dispersion model utilized assumes a collection of M planar wavefronts impinging on the array all originating from the same source and all arriving with the same azimuth angle but with different colatitude angles. This model requires that the phase front corresponding to a given ray arriving at the array center be adequately approximated by a planar surface wherever the phase front contacts the array. If the array length in the azimuthal direction of arrival of the ray is d and the colatitude angle of the ray is θ the distance across the wave front over which planarity should hold is $d \cos\theta$. Each wavefront is characterized by a complex amplitude

$$B_m e^{j\phi_m}, \quad m = 1, 2, \dots, M, \quad (2)$$

at the frequency, f ; the phases and amplitudes are measured at the origin of coordinates, the former relative to an arbitrary reference. B_m will be treated as a random variable, independent of ϕ_m . ϕ_m will be assumed a random variable uniformly distributed in 2π and ϕ_m for different m will be assumed independent. No assumption is made about the dependence among the B_m for different m . The colatitude angle of arrival of a wavefront will be denoted θ_m , $m = 1, 2, \dots, M$. The θ_m will be viewed as nonrandom constants. They may be taken to be equally spaced angular samples. The model employed corresponds to one used by Smith [1] to calculate spatial coherence in a multipath channel.

With the wavefronts arriving at azimuth angle ϕ and the array focused to receive a plane wave from a source at azimuth angle $\frac{\pi}{2}$, i.e. from a source in the y - z plane, and colatitude angle θ_s the total array output becomes

$$A(\phi, \theta_s) = \sum_{m=1}^M \sum_{n=1}^N B_m e^{jk[x_n \sin \theta_m \cos \phi + y_n (\sin \theta_m \sin \phi - \sin \theta_s) + \phi_n]} \quad (3)$$

This is the complex array pattern. The statistical properties of the corresponding array power pattern $|A^2(\phi, \theta_s)|$ will be investigated below.*

* The terminology used here differs somewhat from that given in Urick [2]. We initially calculate an array pattern $|A^2(\phi, \theta)|$ which corresponds to the square of Urick's response function $R^2(\theta, \phi)$ [2, pp. 49-50]. We then determine a normalized mean array pattern which is similar to Urick's "beam pattern" [2, p. 50]. The normalization used here differs however from that used by Urick allowing us to account for loss of coherence across the array. Finally, we define the normalized mean array pattern evaluated on the main beam as the mean power gain.

- [1] P. W. Smith, Jr., "Spatial Coherence in Multipath or Multimodal Channels," Jour. Acoustic. Society of America, Vol. 60, No. 2, August 1976, pp. 305-310.
- [2] R. J. Urick, Principles of Underwater Sound, McGraw Hill Book Co., Second Edition, 1975.

STATISTICAL PROPERTIES OF ARRAY POWER PATTERN

The mean value of the array power pattern is given by

$$\begin{aligned} \langle |A^2(\phi, \theta_s)| \rangle &= \sum_{m_1=1}^M \sum_{m_2=1}^M \sum_{n_1=1}^N \sum_{n_2=1}^N \langle B_{m_1} B_{m_2} \rangle \\ &\quad \langle e^{jk[x_{n_1} \sin\theta_m - x_{n_2} \cos\theta_m + y_{n_1} \cos\phi - y_{n_2} (\sin\theta_m \sin\phi - \sin\theta_s)]} \rangle \\ &\quad \langle e^{j(\phi_{m_1} - \phi_{m_2})} \rangle \end{aligned} \quad (4)$$

The expectation $\langle e^{j(\phi_{m_1} - \phi_{m_2})} \rangle = 1$ when $m_1 = m_2$ and is zero otherwise so that (4) is

$$\begin{aligned} \langle |A^2(\phi, \theta_s)| \rangle &= \sum_{m=1}^M \sum_{n_1=1}^N \sum_{n_2=1}^N \langle B_m^2 \rangle \langle e^{jk(x_{n_1} - x_{n_2}) \sin\theta_m \cos\phi} \\ &\quad \cdot e^{jk(y_{n_1} - y_{n_2}) (\sin\theta_m \sin\phi - \sin\theta_s)} \rangle \end{aligned} \quad (5)$$

Using the assumption that the element positions are independent random vectors (5) is written

$$\begin{aligned} \langle |A^2(\phi, \theta_s)| \rangle &= \sum_{m=1}^M \langle B_m^2 \rangle [N + \sum_{n_1=1}^N \sum_{n_2=1}^N \langle e^{jk(x_{n_1} - x_{n_2}) \sin\theta_m \cos\phi} \\ &\quad \cdot e^{jk(y_{n_1} - y_{n_2}) (\sin\theta_m \sin\phi - \sin\theta_s)} \rangle \\ &= \sum_{m=1}^M \langle B_m^2 \rangle [1 + \left| \sum_{n=1}^N \langle e^{jk[x_n \sin\theta_m \cos\phi + y_n (\sin\theta_m \sin\phi - \sin\theta_s)]} \rangle \right|^2] \\ &\quad - \sum_{n=1}^N \left| \sum_{m=1}^M \langle e^{jk[x_n \sin\theta_m \cos\phi + y_n (\sin\theta_m \sin\phi - \sin\theta_s)]} \rangle \right|^2 \end{aligned} \quad (6)$$

The expectations inside the brackets are two dimensional characteristic functions of the random vectors (x_n, y_n) . Assuming all elements to have identically distributed location vectors and denoting

$$\langle e^{jk[x_n \sin\theta_m \cos\phi + y_n (\sin\theta_m \sin\phi - \sin\theta_s)]} \rangle = \Phi_{xy}(\theta_m, \phi, \theta_s) \quad (7)$$

then

$$\langle |A^2(\phi, \theta_s)| \rangle = \sum_{m=1}^M \langle B_m^2 \rangle [N + (N^2 - N) |\Phi_{xy}(\theta_m, \phi, \theta_s)|^2] \quad (8)$$

At this point we specialize the distribution of the location vectors. We assume the effect of the forces tending to scatter the array elements to be modeled by a two dimensional random walk with independent increments along the coordinate axes. (x_n, y_n) will, after a time, be distributed according to a two dimensional random variable approaching a normal with variance along x and y given by σ_x^2 and σ_y^2 respectively. In this case

$$\Phi_{xy}(\theta_m, \phi, \theta_s) = e^{-[\sigma_x^2 k^2 \sin^2 \theta_m \cos^2 \phi + \sigma_y^2 k^2 (\sin \theta_m \sin \phi - \sin \theta_s)^2]/2} \quad (9)$$

The model chosen accounts for element diffusion but ignores drift components which are sure to be present. The assumption is implied that translation of the entire array will not seriously affect array response when attempting to focus on a distant target if the translation is small. We point out that σ_x^2 and σ_y^2 are functions of time; as the array ages these parameters, which are a measure of the size of the array, will grow.

If we were to suppose that initially the elements are close to one another so that

$$\Phi_{xy}(\theta_m, \phi, \theta_s) = 1 \quad (10)$$

then

$$\langle |A^2(\phi, \theta_s)| \rangle = N^2 \sum_{m=1}^M \langle B_m^2 \rangle = N^2 B^2 \quad (11)$$

$B^2 = \sum \langle B_m^2 \rangle$ is the mean square value of the total signal arriving at the array center and (11) represents the power delivered by the array when it is sufficiently small not to be defocused by the multipath. It is useful to normalize $|A^2(\phi, \theta_s)|$ by $N^2 B^2$. We denote this random variable,^{*}

$$\Gamma = \frac{|A^2(\phi, \theta_s)|}{N^2 B^2} \quad (12)$$

the normalized array power gain. Its mean value is

$$\langle \Gamma \rangle = \frac{1}{N} + (1 - \frac{1}{N}) \sum_{m=1}^M \frac{\langle B_m^2 \rangle}{B^2} e^{-\sigma_x^2 k^2 \sin^2 \theta_m \cos^2 \phi} e^{-\sigma_y^2 k^2 (\sin \theta_m \sin \phi - \sin \theta_s)^2} \quad (13)$$

*An alternative normalization is given by

$$\Gamma_1 = \frac{|A^2(\phi, \theta_s)|}{N^2 \left| \sum_{m=1}^M B_m e^{j\phi_m} \right|^2}$$

This random variable approaches the constant unity when the array shrinks to small size. Γ as given by (12) approaches a random variable whose mean is unity. In Γ_1 , the variability associated with the total arriving signal power at the array center has been removed by the normalization and the quantity is more nearly representative of the effect of array defocusing. Γ_1 is however a more difficult quantity with which to work. We have therefore settled on Γ in which the normalization is done with a constant.

As a final step we define

$$\beta(\theta_m) \Delta\theta = \frac{\langle B_m^2 \rangle}{B^2} \quad (14)$$

that is, we suppose the continuum of possible signal arrival angles to be quantized into increments $\Delta\theta$ and that the fractional power obtained from the m th increment is $\beta(\theta_m)\Delta\theta$. By allowing the increments to become small (13) will be approximated by an integral as follows:

$$\langle r \rangle = \frac{1}{N} + \left(1 - \frac{1}{N}\right) \int \beta(\theta) e^{-\sigma_x^2 k^2 \sin^2 \theta \cos^2 \phi - \sigma_y^2 k^2 (\sin \theta \sin \phi - \sin \theta_s)^2} d\theta \quad (15)$$

$\langle \Gamma \rangle$ as given by (15) is shown evaluated in Figure 2 for the case $\sigma_x^2 = \sigma_y^2 = \sigma^2$ with $\sigma k / 2\pi = \sigma/\lambda$, a family parameter, given by 5, 10, 20, and 40 wavelengths. The power density $\beta(\theta)$ is assumed uniformly distributed over $\pm 10^\circ$ relative to the horizontal. The array is assumed focused for a source in the plane of the array, that is $\theta_s = 90^\circ$, and at an azimuth angle of 90° .

Of particular interest is the magnitude of the normalized mean array pattern evaluated on the main beam as a function of array size. We refer to this quantity as the power gain Γ_o . Its mean value, $\langle \Gamma_o \rangle$, is obtained from (13) or (15) evaluated at $\phi = 90^\circ$. Thus using the discrete ray model we have

$$\langle r_o \rangle = \frac{1}{N} + \left(1 - \frac{1}{N}\right) \sum_{m=1}^M \frac{\langle B_m^2 \rangle}{B^2} e^{-\sigma^2 k^2 (\sin \theta_m - \sin \theta_s)^2} \quad (16)$$

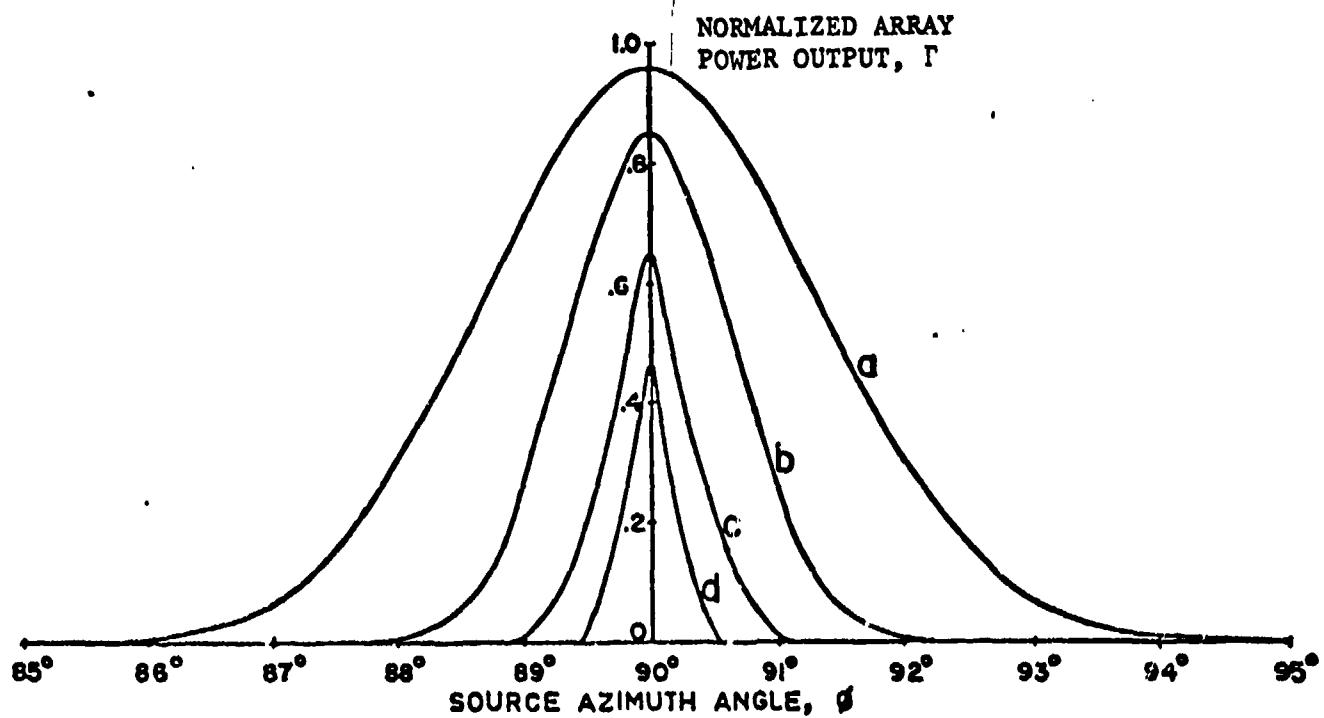


FIGURE 2. PATTERN OF NORMALIZED MEAN ARRAY POWER GAIN.

- (a) $\sigma/\lambda = 5$
- (b) $\sigma/\lambda = 10$
- (c) $\sigma/\lambda = 20$
- (d) $\sigma/\lambda = 40$

Or, using the continuous approximation for the integral,

$$\langle \Gamma_o \rangle = \frac{1}{N} + \left(1 - \frac{1}{N}\right) \int \beta(\theta) e^{-\sigma^2 k^2 (\sin\theta - \sin\theta_s)^2} d\theta \quad (17)$$

In (16) and (17) the subscript on σ_y^2 has been dropped. (17) has been evaluated numerically as a function of the normalized size variable $\sigma k / 2\pi = \sigma/\lambda$, with the angular distribution of energy arriving, $\beta(\theta)$, as a parameter and θ_s set to zero. $\beta(\theta)$ was assumed uniform over angles $\pm 5^\circ$, $\pm 10^\circ$, and $\pm 20^\circ$ relative to the plane of the array. Results are shown in Figure 3. For distant sources the arrival angles are apt to be within

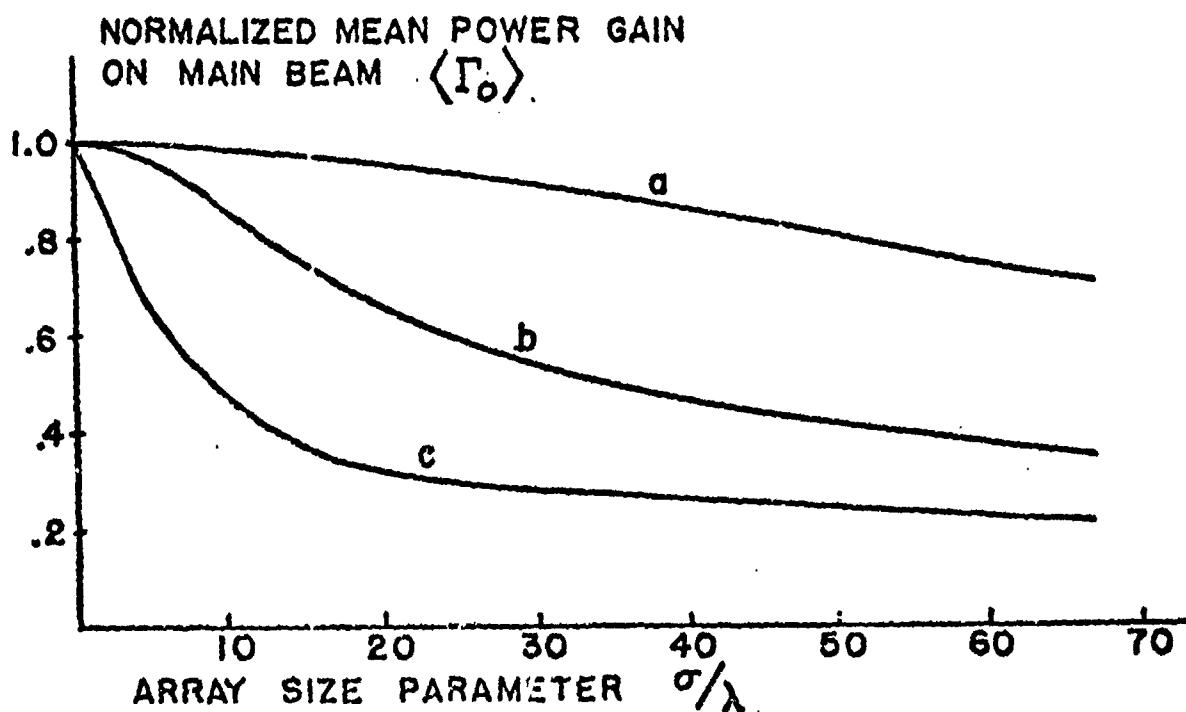


FIGURE 3. | MEAN POWER GAIN AS A FUNCTION OF NORMALIZED
| ARRAY SIZE VARIABLE, σ/λ .

- (a) $\beta(\theta) = 1/10$, $85^\circ < \theta < 95^\circ$
- (b) $\beta(\theta) = 1/20$, $80^\circ < \theta < 100^\circ$
- (c) $\beta(\theta) = 1/40$, $70^\circ < \theta < 110^\circ$

$\pm 10^\circ$. Note that delivered power is reduced to about 1/2 when $\sigma/\lambda = 35$ wavelengths. For sources nearby, bottom and top reflections may result in energy arriving at steeper angles and the $\pm 20\%$ distribution may be viewed as a model suggesting the effect in such a case. Here delivered power is reduced to about 1/2 where $\sigma/\lambda = 8$ wavelengths.

Setting the angle θ_s to zero means focusing the array for signal arrivals in the plane of the array. This is not optimum for signals arriving over a dispersion of latitude angles. To show this we have plotted this mean power gain as a function of the colatitude angle θ_s for the case of $\beta(\theta)$ uniform in $\pm 10^\circ$ around the horizontal and $\sigma/\lambda = 33.3$ wavelengths. The result is shown in Figure 4. The maximum is seen to occur with the beam formed for $\theta_s = 86^\circ$ (by symmetry it will also be maximum for $\theta_s = 94^\circ$).

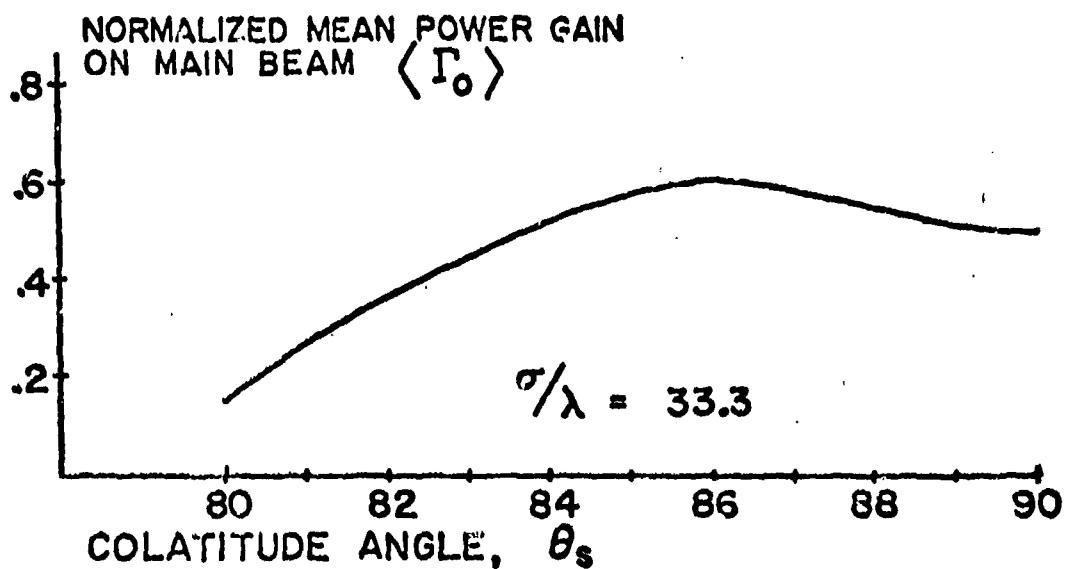


FIGURE 4. MEAN POWER GAIN AS A FUNCTION OF
VERTICAL AIMING ANGLE θ_s .
 $\beta(\theta) = 1/20, 80^\circ < \theta < 100^\circ$

The variance of the power gain will be useful as an indicator of the gain variability. We thus determine $\text{Var } \Gamma_o = (\langle \Gamma_o^2 \rangle - \langle \Gamma_o \rangle^2)$ where $\Gamma_o = |A^2(\frac{\pi}{2}, \theta_s)| / N^2 B^2$. Starting with (3) evaluated at $\phi = \pi/2$ we have

$$A(\frac{\pi}{2}, \theta_s) \equiv A_o = \sum_{m=1}^M \sum_{n=1}^N B_m e^{j[k y_n (\sin \theta_m - \sin \theta_s) + \phi_m]} \quad (18)$$

The fourth moment of the magnitude is given by

$$\begin{aligned} \langle |A_o|^4 \rangle &= \sum_{\substack{m_1, m_2, m_3, n_1, n_2, n_3, \\ m_4=1}}^M \sum_{n_4=1}^N \langle B_{m_1} B_{m_2} B_{m_3} B_{m_4} \rangle \\ &\quad \cdot \langle e^{j(k_{m_1} y_{n_1} - k_{m_2} y_{n_2} + k_{m_3} y_{n_3} - k_{m_4} y_{n_4})} \rangle \\ &\quad \cdot \langle e^{j(\phi_{n_1} - \phi_{n_2} + \phi_{n_3} - \phi_{n_4})} \rangle \end{aligned} \quad (19)$$

We have used the abbreviated notation $k_{m_1} = k(\sin \theta_{m_1} - \sin \theta_s)$. With the ϕ_{n_i} independent uniformly distributed random variables in $(0, 2\pi)$ (19) reduces to

$$\begin{aligned} \langle |A_o|^4 \rangle &= \sum_{\substack{n_1, n_2, n_3, \\ n_4=1}}^N \left\{ \sum_{\substack{m_1, m_3=1 \\ m_1 \neq m_3}}^M \langle B_{m_1}^2 B_{m_3}^2 e^{j[k_{m_1} (y_{n_1} - y_{n_2}) + k_{m_3} (y_{n_3} - y_{n_4})]} \rangle \right. \\ &\quad + \sum_{\substack{m_1, m_2=1 \\ m_1 \neq m_2}}^M \langle B_{m_1}^2 \rangle \langle B_{m_2}^2 \rangle e^{j[k_{m_1} (y_{n_1} - y_{n_4}) + k_{m_2} (y_{n_3} - y_{n_2})]} \\ &\quad \left. + \sum_{m=1}^M \langle B_m^4 \rangle \langle e^{j(k_m (y_{n_1} - y_{n_3} + y_{n_2} - y_{n_4}))} \rangle \right\} \end{aligned} \quad (20)$$

Summation over the n_i , $i = 1, 2, 3, 4$, is now carried out using the assumption that the positions of the different elements, the y_k , $k=1, 2, \dots, N$, are independent. The summation is straightforward although laborious.

Carrying out the steps we can then write

$$\begin{aligned}
 \text{Var } \Gamma_o &= \frac{\langle |A_o^4| \rangle - \langle |A_o^2| \rangle^2}{N^4 B^4} \\
 &= \sum_{m_1=1}^M \sum_{m_2=1}^M \frac{\langle B_{m_1}^2 \rangle \langle B_{m_2}^2 \rangle}{B^4} \cdot \frac{1}{N^4} \left\{ N^2 + N^2(N-1) \left[e^{-\sigma^2 k_{m_1}^2} + e^{-\sigma^2 k_{m_2}^2} \right] \right. \\
 &\quad + 2N(N-1) \left[e^{-\sigma^2 (k_{m_1} + k_{m_2})^2} + e^{-\sigma^2 (k_{m_1} - k_{m_2})^2} \right] \\
 &\quad + 4N(N-1)(N-2) \left[e^{-\frac{\sigma^2}{2}(k_{m_1}^2 + k_{m_2}^2)} \left[e^{-\frac{\sigma^2}{2}(k_{m_1} + k_{m_2})^2} + e^{-\frac{\sigma^2}{2}(k_{m_1} - k_{m_2})^2} \right] \right. \\
 &\quad \left. \left. + \left[2N(N-1)(N-2)(N-3) - N^2(N-1)^2 \right] e^{-\sigma^2 (k_{m_1}^2 + k_{m_2}^2)} \right\} \right. \\
 &\quad + \sum_{m=1}^M \frac{\langle B_m^4 \rangle - 2\langle B_m^2 \rangle^2}{B^4} \cdot \frac{1}{N^4} \left[N(2N-1) + 4N(N-1)^2 e^{-\sigma^2 k_m^2} \right. \\
 &\quad + N(N-1)(N-2)(N-3) e^{-2\sigma^2 k_m^2} \\
 &\quad + 2N(N-1)(N-2) e^{-3\sigma^2 k_m^2} \\
 &\quad \left. \left. + N(N-1) e^{-4\sigma^2 k_m^2} \right] \right. \tag{21}
 \end{aligned}$$

The ratio $\text{Var } \Gamma_o / \langle \Gamma_o \rangle^2$, where the numerator is given by (21) and the denominator by the square of the mean power gain given in (16), is a useful measure of relative variance. The result is cumbersome,

however, and expressions applicable to limiting cases are instructive.

Two such cases are here evaluated assuming the ray amplitudes, B_m , are equal for all m and constant so that $\langle B_m^k \rangle / B^k = (1/M)^{k/2}$, k even.

The first case treated assumes the elements very widely dispersed so that $\sigma_{k_m}^2 \gg 1$ for all rays except ones for which $\sin\theta_m = \sin\theta_s$.

Since $k_m = k (\sin\theta_m - \sin\theta_s)$ a ray along the aiming angle will result in $k_m = 0$. Assuming one ray is along the aiming angle set at $\theta_s = 90^\circ$ we get, using (16) and (21),

$$\frac{\text{Var } \Gamma_o}{\langle \Gamma_o \rangle^2} = \frac{NM^2 + M(2N^2 - 2N - 1) - (2N^2 - N - 1)}{N(M+N-1)^2} * \quad (22)$$

When $M = 1$, there is only 1 ray, and that one along the aiming angle.

There is no multipath and the array will be correctly focused. The ratio above is then zero. When M gets large without bound while N remains finite the ratio approaches unity. This result can be anticipated. For M large the elements, being widely dispersed, see a sinusoid with Rayleigh magnitude and random phase. The ratio in (22) in that case is that of the variance and squared mean of an exponential random variable for which this ratio is unity. If M is held finite while N is allowed to increase without bound the ratio tends to zero. This result arises because the element outputs caused by the one ray along the aiming angle are coherently combined by the array. The random component contributed by the rays off-axis add up non-coherently at the array output. The latter are the variance producing components. But as N increases

*This result assumes also that $\sin k_m_1 \neq \sin k_m_2$ for all $m_1 \neq m_2$. Should there be rays arriving symmetrically relative to the horizontal there will be some $\sin k_m_1 = \sin k_m_2$ for $m_1 \neq m_2$. In such a case some additional terms will be required from (21) and (22) may exceed unity.

The ratio of the non-coherent components tends to zero.

We point out that if no ray comes in at an angle sufficiently close to θ_s to make a significant coherent contribution then $\text{Var } \Gamma_o / \langle \Gamma_o \rangle^2$ approaches $(1-1/MN)$. Now the ratio approaches unity with increasing M or N as one would expect.

The second case treated is one for which the element locations are reasonably compact, say within $\sigma/\lambda = 10$, and N is large. In this case, if ray arrivals are within $\pm 10^\circ$ of the vertical focusing angle θ_s , $\sigma^2 k_m^2$ is small and $e^{-2\sigma^2 k_m^2}$ is close to unity (it is 0.748 for $\theta = 10^\circ$). The predominant terms in (21) are then those of highest degree in N. Extracting those terms we have

$$\begin{aligned} \text{Var } \Gamma_o &\doteq \sum_{m_1=1}^M \sum_{m_2=1}^M \frac{\langle B_{m_1}^2 \rangle \langle B_{m_2}^2 \rangle}{B^4} e^{-\sigma^2 (k_{m_1}^2 + k_{m_2}^2)} \\ &+ \sum_{m=1}^M \frac{\langle B_m^4 \rangle - 2\langle B_m^2 \rangle^2}{B^4} e^{-2\sigma^2 k_m^2} \end{aligned} \quad (23)$$

In (23) we have set such factors as $(N-1)(N-2)(N-3)/N^3$ to unity in conformity with specification that N be large. Again now with B_m a constant and equal for all m

$$\begin{aligned} \text{Var } \Gamma_o &\doteq \frac{1}{M^2} \left[\left(\sum_{m=1}^M e^{-\sigma^2 k_m^2} \right)^2 - \sum_{m=1}^M e^{-2\sigma^2 k_m^2} \right] \\ &\approx \langle \Gamma_o \rangle^2 \left[1 - \frac{\sum_{m=1}^M e^{-2\sigma^2 k_m^2}}{\left(\sum_{m=1}^M e^{-\sigma^2 k_m^2} \right)^2} \right] \end{aligned} \quad (24)$$

$\langle \Gamma_o \rangle^2$ is obtained from (16) using the assumptions pertinent to this case. The ratio inside the bracketed factor in (24) is $1/M$ for $\sigma = 0$. For $\sigma = 0$ (24) is identically the variance of the squared amplitude of the sum of M equal amplitude sinusoids with independent phases, all uniformly distributed in 2π ; this result for $\sigma = 0$ simply reflects the variability of the incoming total signal magnitude at the array center. It is interesting to observe that if B_m were Rayleigh distributed, $\text{Var } \Gamma_o / \langle \Gamma_o \rangle^2$ would be unity for all M and for all σ/λ for which (23) is valid.

To convey some idea of how the variance changes with the array size parameter σ/λ , and to provide some results without approximations the ratio $\text{Var } \Gamma_o / \langle \Gamma_o \rangle^2$ was calculated for some representative cases using (16) and (21) as they stand. Table 1 shows these results for the case of M rays, $M = 3, 5, 11$, and 21 , the rays arriving at equally spaced angles in an interval of $\pm 10^\circ$ relative to the horizontal. The beam pointing angle was set along the horizontal ($\theta_s = 90^\circ$) and in the direction of the source. The number of sensors N was taken to be 31 . It is worth noting that for $M = 3$, $N = 31$ in the limiting expression (22) ($\sigma/\lambda \rightarrow \infty$), $\text{Var } \Gamma_o / \langle \Gamma_o \rangle^2 = 0.1175$ only slightly below the calculated value in Table 1 for $M = 3$, and $\sigma/\lambda = 60$.

There may be another phenomenon responsible for the trend of the calculated values in Table 1. With $M > 1$ the signal power at the array center is a random variable as a consequence of the random phases of the incoming rays. The ratio of $\text{Var } \Gamma_o / \langle \Gamma_o \rangle^2$ is, as pointed out below (24), given by $(1-1/M)$. This is exactly the value in Table 1 for

$\sigma/\lambda = 0$. As σ/λ increases the ratio $\text{Var } \Gamma_o / \langle \Gamma_o \rangle^2$ decreases, partly for the reason discussed under (24), and perhaps also because the signal amplitude which varies with position is averaged by the array.

$\frac{\sigma}{\lambda}$	M			
	3	5	11	21
0	0.67	0.80	0.91	0.95
5	0.66	0.80	0.91	0.95
10	0.62	0.78	0.91	0.95
15	0.42	0.73	0.90	0.95
20	0.20	0.70	0.88	0.95
40	0.12	0.64	0.84	0.92
60	0.12	0.47	0.81	0.91

TABLE 1. RATIO OF VARIANCE TO SQUARED MEAN OF ARRAY OUTPUT AS A FUNCTION OF ARRAY SIZE, σ/λ , AND NUMBER, M, OF EQUAL AMPLITUDE RAYS; RAYS ARE ASSUMED EQUALLY SPACED IN AN INTERVAL $\pm 10^\circ$ FROM HORIZONTAL.

CONCLUSION

Mean power pattern, mean main beam gain and main beam gain variance have been determined for an acoustic array of widely scattered submerged elements. The elements are assumed to be organized to accept a plane wave but the array sees a multipath field typical of the acoustic field at a great distance from a source in the deep ocean. The results obtained show the diminution of array effectiveness as the array size grows; for a typical case where the range of latitude angles of the arriving signal is 10° above and below the horizontal, the mean power gain falls 3 dB when the element spread as measured by the element position standard deviation is 35 wavelengths.

In earlier work Smith [1] calculated the normalized coherence magnitude as a function of horizontal separation in a long-range transmission channel with bigradient sound speed profile. In particular he explicitly obtains the coherence distance for 50% coherence for a source on axis and a receiver close to the channel edge. It turns out to be 46 wavelengths if the difference in sound speed between the axis and the receiver location is 20m/sec. The range of vertical angles of arrival is $\pm 9.36^\circ$ relative to the horizontal at the receiver and the energy density, obtained from earlier results, is assumed by him to be uniform over the range of arrival angles. This situation is roughly the same as that used to arrive at the 35 wavelength figure mentioned above. The two results are interestingly similar. This ought to come as no surprise; since the coherence distance corresponds to the array dimension useful for coherent combination of the spatially sampled field. The crucial factor in determining the coherence distance is the range of angles of arrival. To

see this intuitively, imagine two multipath rays, one horizontal and the other smaller in amplitude and at θ° to the horizontal. The phase ψ of the resultant vs horizontal distance, x , is expressible as [3]

$$\psi(x) = kx + \phi(x)$$

where k is the wave number. $\phi(x)$, is periodic with period $2\pi/k(1-\cos\theta) = \lambda/(1-\cos\theta)$. With $\theta = 10^\circ$ the period is about 66λ . This is not too far different from the 2σ range (= 70 wavelengths) for an array power gain of 0.5, or from the 46 wavelengths obtained by Smith for coherence distance for 0.5 normalized coherence.

[3] F. Haber, "Phase Variations with Position in an Underwater Multipath Environment and its Effect on Array Pattern," Valley Forge Research Center Quarterly Progress Report No. 24, University of Pennsylvania, the Moore School of Electrical Engineering, Philadelphia, PA 19104, pp. 20-30.

ACKNOWLEDGMENTS

I am indebted to Mr. William J. Graham for his review of part of this work and for his suggestions for improvement, to Mr. DeYuan Ho for his help with the numerical evaluations, and to the U. S. Office of Naval Research for its support on Contract N00014-77-C-0252. I am also indebted to the personnel of the Office of Naval Research and of the Naval Ocean Systems Center, San Diego, California, for providing a system description which was used as a guide to practical values in this paper.

-- 11 - TITLE: (U) RESEARCH IN DISTRIBUTED ARRAYS
-- 1 - AGENCY ACCESSION NO: DN775290
-- 2 - DATE OF SUMMARY: 12 NOV 80
-- 39 - PROCESSING DATE (RANGE): 30 NOV 80
-- 8 - SECURITY OF WORK: UNCLASSIFIED
-- 12 - S + T AREAS:
-- 000200 ACOUSTICS
-- 000100 ACOUSTIC DETECTION
-- 21E - MILITARY/CIVILIAN APPLICATIONS: MILITARY
-- 10A1 - PRIMARY PROGRAM ELEMENT: 82711N
-- 10A2 - PRIMARY PROJECT NUMBER: F11121
-- 10A2A - PRIMARY PROJECT AGENCY AND PROGRAM: F11121
-- 10A3 - PRIMARY TASK AREA: RF11121810
-- 10A4 - WORK UNIT NUMBER: NR-129-102
-- 17A1 - CONTRACT/GRANT EFFECTIVE DATE: MAR 77
-- 17A2 - CONTRACT/GRANT EXPIRATION DATE: FEB 80
-- 17B - CONTRACT/GRANT NUMBER: N00014-77-C-0252
-- 17C - CONTRACT TYPE: COST TYPE
-- 17D2 - CONTRACT/GRANT AMOUNT: \$ 25,000
-- 17E - KIND OF AWARD: EXT
-- 17F - CONTRACT/GRANT CUMULATIVE DOLLAR TOTAL: \$ 130,000
-- 19A - DOD ORGANIZATION: OFFICE OF NAVAL RESEARCH 222

-- 1 OF 1
-- 11 - TITLE: (U) RESEARCH IN DISTRIBUTED ARRAYS
-- 1 - AGENCY ACCESSION NO: DN775290
-- 2 - DATE OF SUMMARY: 12 NOV 80
-- 39 - PROCESSING DATE (RANGE): 30 NOV 80
-- 8 - SECURITY OF WORK: UNCLASSIFIED
-- 12 - S + T AREAS:
-- 000200 ACOUSTICS
-- 000100 ACOUSTIC DETECTION
-- 21E - MILITARY/CIVILIAN APPLICATIONS: MILITARY
-- 10A1 - PRIMARY PROGRAM ELEMENT: 82711N
-- 10A2 - PRIMARY PROJECT NUMBER: F11121
-- 10A2A - PRIMARY PROJECT AGENCY AND PROGRAM: F11121
-- 10A3 - PRIMARY TASK AREA: RF11121810
-- 10A4 - WORK UNIT NUMBER: NR-129-102
-- 17A1 - CONTRACT/GRANT EFFECTIVE DATE: MAR 77
-- 17A2 - CONTRACT/GRANT EXPIRATION DATE: FEB 80
-- 17B - CONTRACT/GRANT NUMBER: N00014-77-C-0252
-- 17C - CONTRACT TYPE: COST TYPE
-- 17D2 - CONTRACT/GRANT AMOUNT: \$ 25,000
-- 17E - KIND OF AWARD: EXT
-- 17F - CONTRACT/GRANT CUMULATIVE DOLLAR TOTAL: \$ 130,000
-- 19A - DOD ORGANIZATION: OFFICE OF NAVAL RESEARCH 222

-- 19B - DOD ORG. ADDRESS: ARLINGTON, VA 22217
-- 19C - RESPONSIBLE INDIVIDUAL: BOYER, G L 222
-- 19D - RESPONSIBLE INDIVIDUAL PHONE: 202-696-4205
-- 19U - DOD ORGANIZATION LOCATION CODE: 5110
-- 19S - DOD ORGANIZATION SORT CODE: 35832
-- 19T - DOD ORGANIZATION CODE: 285250

-- 20A - PERFORMING ORGANIZATION: UNIV OF PENNSYLVANIA, MOORE SCHOOL OF
-- ELECTRICAL ENGINEERING
-- 20B - PERFORMING ORG. ADDRESS: PHILADELPHIA, PA 19174
-- 20C - PRINCIPAL INVESTIGATOR: HABER, F
-- 20D - PRINCIPAL INVESTIGATOR PHONE: 215-243-8104
-- 20F - ASSOCIATE INVESTIGATOR (1ST): HABER, F
-- 20G - ASSOCIATE INVESTIGATOR (2ND): LIM, T
-- 20U - PERFORMING ORGANIZATION LOCATION CODE: 4201
-- 20V - PERF. ORGANIZATION TYPE CODE: 1
-- 20S - PERFORMING ORG. SORT CODE: 31456
-- 20T - PERFORMING ORGANIZATION CODE: 237000
-- 22 - KEYWORDS: (U) RANDOM ARRAYS , (U) DISTRIBUTED ARRAYS,(U)
SONOBUDY ,(U) PASSIVE SONAR ,
-- 37 - DESCRIPTORS: (U) DISTRIBUTION,(U) GAIN ,(U) PASSIVE SONAR ,
(U) SONOBUDYS ,(U) UNDERWATER SOUND ,(U) ACOUSTIC COMPATIBILITY ,
(U) UNDERWATER ACOUSTICS ,(U) *ACOUSTIC ARRAYS ,(U) *ACOUSTIC
DETECTION,(U) ACOUSTIC TRACKING ,(U) EFFECTIVENESS ,(U)
-- ELECTROMAGNETIC COMPATIBILITY ,
-- 23 - TECHNICAL OBJECTIVE: (U) TO DEVELOP TECHNIQUES IN SIGNAL ANALYSIS
TO MAXIMIZE ACHIEVABLE ARRAY GAIN OF A RANDOM SONOBUDY ARRAY.
-- 24 - APPROACH: (U) INVESTIGATE PHASE DECORRELATION EFFECT ON ARRAY GAIN
AND WAYS TO OVERCOME THESE EFFECTS.
-- 25 - PROGRESS: (U) EARLIER WORK FOCUSED ON METHODS OF LOCALIZING ELEMENTS
OF A RANDOM ARRAY. THEORY HAS BEEN DEVELOPED. VALLEY FORGE RESEARCH
CENTER QUARTERLY PROGRESS REPORT, FEB. 1979 (U).
-- 13 - WORK UNIT START DATE: MAR 77
-- 14 - ESTIMATED COMPLETION DATE: CO